TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

No. 285

CALCULATION OF THE HULL AND OF THE CAR-SUSPENSION SYSTEMS OF AIRSHIPS.

By R. Verduzio.

From "Rendiconti Tecnici," March 15, 1924.

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I N D E X

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Flexible Band for Tracing.

A thin, flexible, elastic metal band of constant section, bent, as shown in Fig. 1, by the action of two forces f applied to the extremities and of two moments m, undergoes at any point P, a bending moment of

$$M = m + f\ y' = f(h_1 + y) = f\ y \tag{28}$$

in which y' indicates the height of point P above point A; h_1, a constant height; and y the theoretical height at which the force f should act, if it alone were to create the bending moment M at point P.

By putting

$$\frac{E\ J}{r} = M \tag{29}$$

and substituting in equation (28) the value of M given by equation (29), we obtain:

$$r\ y = \frac{E\ J}{f} = \text{const.} \tag{30}$$

which is equation (9) of the lythenary.

This signifies that the band takes on the shape of the imponderable hull.

By utilizing this property, Col. Crocco built a device for tracing the shape of hulls. (See Crocco: "Rendiconti Esperienze e Studio dello Stabilmento Costruzioni Aeronautiche," Vol. III, 1914.)

"It is based primarily on the observation that equation (29) is not dependent on the particular form and flexibility of the band used, because for similar forms, the force f, which it is necessary

to introduce, is itself proportional to the product E J. It is therefore sufficient to employ any kind of band or strip or wire, provided it is homogeneous and of constant section and highly flexible in the sense that in bending it, the elastic limit of the material be not exceeded, its dimensions and modulus of elasticity counting for little.

"Secondly, the device is based on the principle that it is unnecessary to know the force f, but only initially, the position of the plane of equipressure, that is, the distance y, measured from a well-determined point of the curve, where said force is acting.

"In view of this, the device was obtained by connecting to the ends of the flexible band two wooden rods or metal tubes fitted with adjustable connections permitting the angle F, between the direction of the band at the point of attachment and that of the rod at the same point, to be varied. Furthermore, two runners c, united by an articulated joint s, are fitted to the two rods (Fig. 2).

"A few small pulleys r, support the weight of the device and enable smooth running. The flexible band is graduated in millimeters. The weight of the device may also be sustained by hinging the joints P and Q to fixed supports.

"After the position of the rod a, is fixed, the next important operation is setting the two rods a and b, perfectly parallel, which is accomplished by adjusting the two attachments.

"The procedure is as follows:

1) The attachments are fixed at a distance corresponding, in the scale of the draft, to the desired length of the hull section, which

is included between the two fixed points A B, through which the curve must pass. Otherwise, two indexes of any kind are marked at the ends of said length, measured on the graduated band.

2) The positions of the points P and Q, located on the plane of equipressure and of which the distance from each other is known, as well as their distance respectively from points A and B, are fixed on the rods.

3) The screws of the attachments are adjusted until the rods are parallel and the curve passes through the fixed points A and B.

"In this way, the device remains in equilibrium as a result of the single force (unknown) acting between points P and Q, and thus corresponds exactly to the equations established."

Tracing of the Gores.

The surface of an airship hull can not be developed into a plane. Therefore, for the purposes of construction of a hull, said surface must be replaced by other surfaces which may be developed and which so closely approach the original surface that, when the hull is inflated, the shape is almost perfect.

The surface lines will meet in curves corresponding to the longitudinal seams of the fabric. It is evident, therefore, that the length of two adjacent edges of two partial surfaces in contact with each other, must be equal.

The methods for determining the resolution of the hull into elements capable of development, are:

a) Resolution into gores along the meridians, that is, along longitudinal geodetic lines;

b) Resolution into inscribed, circumscribed, or median cones, that is, into parallel zones;

c) Resolution into transverse geodetic zones.

The method of development along longitudinal geodetic lines consists in tracing on the hull a certain number of meridians, possibly at equal angular distances from each other, and passing through the points of variation of form of the section. In this way, the same number of gores is obtained as the number of meridians traced. Each gore of the hull is then replaced by the surface which is obtained by shifting longitudinally a straight line, which, being tangential to the hull and invariably parallel to itself, runs along the meridian at right angles. Consequently, for each gore, a surface, susceptible of development into a plane, is obtained, possessing almost in every case an axis of symmetry which is equal to the length of the meridian, said surface being bounded by two curves.

If the meridian curve of the hull has the equation

$$y = f\,(x)$$

and if a certain portion of the hull section has n identical gores, the semi-width h, of each gore, in correspondence with the parallel of the abscissas x and the ordinates y, will invariably have the form

$$h = \frac{A}{n}\, f\,(x) \tag{37}$$

in which A is a numerical factor, depending on the form of the

contour of the section on which the gores must lie, and n is the smallest integral number, as defined by

$$L \geq \frac{2A}{n} f\ (x_1) \tag{38}$$

in which L is the width of the fabric to be employed and x_1 the value of x corresponding to the main cross-section.

The abscissa S, corresponding to a given value h of the ordinate of the gore, is

$$S = \int_0^S ds = \int_0^x \sqrt{1 + f'(x)^2}\, dx \tag{39}$$

The method indicated above, for calculating ordinates and abscissas of every gore, is very simple and may always be applied to hulls, the meridians of which may be formulated mathematically, and also to hulls the meridians of which can be determined experimentally. In the latter case, it is sufficient to substitute y for $f(x)$ and the $+$ sign for the $\int$ in the above and following formulas.

In progressing from the main cross-section toward the bow and stern, the value of y decreases and, consequently, also that of n. In order, therefore, to avoid too close seams at the ends of the hull, it is expedient after the section, the abscissa of which is x, is reached, to decrease the number of the seams, so that

$$a\ L \geq b\ \frac{2A}{n}\ (x) \tag{40}$$

in which a and b are integral numbers.

Suitable solutions are obtained by assuming $a = 2$ and $b = 3$, or $a = 1$ and $b = 2$, and by reducing the seams at the ends of the hull by using the combination of the two solutions proposed.

The reduction in the number of seams (from a number of gores n to a smaller number) is effected, so that the seams do not all coincide with the same parallel. It is better that they be given an alternate arrangement, as shown in Fig. 3, where the gores have been traced with equal width for the sake of clearness.

In passing from a given number of gores to a smaller number, we find that, theoretically, their tracing, as indicated above, is not suitable, because the fabric would overlap at certain points, while it would be totally lacking at other points. Hence the conception that the tracing should be made by substituting, for the ordinates h and for the axis of the abscissas s, segments of a circle h in length, the center of which, however, is located on the axis of the s, with a radius equal to the length of the tangents to the meridian curve included between the point of contact and the longitudinal axis of the airship.

Theoretically, this is exact, but it would entail considerable difficulty in drafting. On the other hand, this disadvantage is not encountered in practice and, consequently, the above-mentioned method of tracing holds.

Around the drawing of the gore, described above, it is necessary to leave an additional strip, one-half the width of the fabric, for the seam.

In practice, the tracing of the gores is made by tin soldering a steel wire 0.5 to 1 mm in diameter to a piece of zinc sheet of suitable dimensions, along the curves determined as above. The fabric is then stretched on the metal sheet prepared in this way. By using a smoke-covered roller, the fabric is then marked in correspondence with the curve along which the fabric has to be cut and sewed.

The abscissas S for the composition of the various gores and parts, as well as the other reference points considered necessary, are also marked on the zinc sheet.

The method of development into parallel zones consists in tracing on the surface of the hull a certain number of parallels and in considering the fabric rings included between two consecutive parallels as susceptible of development. This is tantamount to substituting for the hull surface the surfaces of a number of cones, the vertexes of which are located on the longitudinal axis of the airship at the points where said axis is intersected by the straight line joining the points in which the two parallels under consideration cut the meridian, or at the points where said axis is intersected by the tangents to the median element of the meridian included between the two parallels, or again, where it is intersected by a straight line between the two former lines. Consequently, three different methods are available for this development.

The zone of conical surface thus obtained may be developed into a plane. The two parallels bounding this zone, are circles or

curves easy of determination and depending on the shape of the section.

The parallels must be selected sufficiently close to one another so that the arc of the meridian included between them may be replaced by a segment of a straight line.

This method is advantageous, when the hull is wholly a figure of revolution.

If the equation of the meridian is

$$y = f\ (x)$$

the angle a between the axis of rotation and the tangent to the meridian in a point the abscissa of which is x, is

$$\tan a = f^1\ (x)$$

the angle a between the axis of rotation and the chord of the arc of the meridian, as characterized by the points x and $x + dx$, is:

$$\tan a = \frac{f\ (x + dx) - f\ (x)}{dx}$$

The length of the generatrix of the cone included between the vertex and the generatrix of contact on the hull, or the parallel corresponding to x, will be:

$$R = \frac{f\ (x)}{\sin a} \tag{41}$$

and in developing the cone in a plane, R is the radius of the circle with which the median parallel or the parallel of intersection of the cone with the hull surface (that is, the internal circle of the extended zone) will be developed.

In the first case, the semi-width of the zone will be

$$h = r \tan e \qquad (42)$$

where r is the radius of the meridian curve at the point of contact and a the angle measured at the center of the osculatory circle corresponding to the external angles of the arc of the meridian between the two successive parallels.

In the second case, the width of the zone will be:

$$3\ h = \frac{\Delta x}{\cos a} \qquad (43)$$

The problem is thus reduced merely to tracing arcs of circles.

The tracing of the arcs for the outline of the elements nearest the pole, entails no special difficulty, if an extension compass is used. But when the radius exceeds 4 meters, it is impossible to continue with this system, both on account of practical difficulties and of the inevitable inaccuracy. Recourse is therefore had to an indirect method based on the principle that the geometrical locus of all points which, joined to two fixed points, form a constant angle, is a circle. Consequently, if it is desired to trace the arc $A_1B_1C_1$ of radius R and underlying chord $2\ c_1$, it is sufficient to employ two rods, one passing constantly through A_1 and the other through B_1, and determine the point of their intersection so that OC is perpendicular to the chord. By moving the point of intersection, while maintaining the rods passing constantly through A_1B_1, said point will describe the desired arc.

This procedure, however, would necessitate varying the distance

between the points A_1 and B_1, for each arc, and modifying the inclination of the rods so that C is such that $y_1 = R - \sqrt{R^2 - C_1^2}$.

If, instead of taking $A_1B_1 = 2c_1$ as the basis, we take $AB = 2x$ of any length, we are able to trace the same arc by taking

$$y = R - \sqrt{R^2 - X^2}.$$

It is therefore, obvious that the operation may be simplified by keeping the basis 2x constant and varying only the inclination of the two rods according to the y calculated. According to the principles set forth above, a practical device for drawing arcs of great radius has been constructed (Fig. 4).

This device comprises:

a) A board ABCDEF with a graduated reference line $f_1 f_2$ traced in its median axis. Along the side EF another double graduation is marked, zero being located at the point of intersection with the $f_1 f_2$ line;

b) Two movable rods (a_1, a_2) carrying a tracer at the point of crossing;

c) Two roller supports (r_1, r_2) fixed to the board on a parallel to the side EF and at a predetermined distance 2x from each other;

d) A square.

By means of straight lines passing through its center, the zone developed, is divided into a number of parts which are included in the width of the fabric. The geometrical figures thus obtained are curvilinear isosceles trapezia.

It is advisable to determine the maximum length to be given to one of the parallel sides of the trapezium. This length L is deter-

mined by the fact that the radial f of the arc forming the minor parallel side, increased by $2h$, must be equal to the width of the fabric, that is,

$$f = L - 2h, \tag{44}$$

and the length $2c_1$ of the chord of an arc $2F$, the radius of which is R, is given by

$$c_1 = R \sin F = R \sqrt{1 - \left(\frac{R - f}{R}\right)^2} \tag{45}$$

Consequently, the problem is completely solved.

In sewing the different conical rings together, it is better for the seams corresponding to the straight sides of the trapezia to be in zig-zag arrangement. In this way a stronger and easier construction is obtained.

The method of development into geodetic zones consists in tracing on the surface a certain number of geodetic curves arranged so that they are tangential to the parallels at the point corresponding to the extreme upper point of their vertical diameter. This fact causes the geodetic lines to be symmetrical on both sides with respect to the vertical diametral plane of the airship. The geodetic line of the main cross-section coincides with the parallel. The surface included between the geodetic lines will be replaced by the developed surfaces obtained as the geometrical locus of the curves of the meridian arcs included between two successive geodetic lines.

Determination of the Compartments.

Diaphragms.

The volume of a compartment must be such that, assuming it to deflate completely, the airship will continue to float. This means that the lifting force corresponding to one compartment must be less than the lightening which may be obtained in a given airship, taking into account the ballast, the lifting action of the entire airship and the positive thrust given by the ascensional propellers, in case there are any.

The number of compartments is thereby determined. Furthermore, if the axis of the hull is to remain horizontal or nearly so, it will be necessary to let a certain amount of gas escape from the compartment which is the farthest away from (and on the opposite side of) the center of gravity with respect to the compartment which is deflating. However, in this way, navigation would be hampered by a series of drawbacks. In general, it is not best to deflate voluntarily the extreme compartments. All the compartments could be reduced to one-half the volumes determined above, but the subdividing would be too great and the weight of the hull excessive. This is remedied by making the extreme compartments about half the size of the central compartments. By skillful operation of the gas valves it is then possible to keep the axis of the airship nearly horizontal. Having determined the number of compartments and the law of variation, the latter is plotted on a straight line paral-

lel to the ordinates of the integral curve of the volumes. By means of lines parallel to the axis of the abscissas, drawn through the points indicating the law of variation, it is possible to determine on the integral curve a number of points corresponding to the positions of the diaphragms. These, however, may be slightly shifted.

Let us now consider the shape and the construction of the diaphragms. In discussing the cross-section of the hull, we stated that this varies according to the internal pressure. The diaphragms, which are continuous walls between one gas compartment and the next, must permit such variation of form, so as to prevent the hull from developing a number of humps and hollows from bow to stern.

Hence the necessity for the diaphragms to have such dimensions that the sections of the hull for various degrees of inflation at zero pressure or at the maximum pressure supportable by the valves, are contained in the surface of the diaphragm. It is necessary, however, that the dimensions be the smallest possible, in order to prevent any considerable shifting of the central part of the diaphragm and consequently displacement of large quantities of gas in the bag. Notwithstanding this last consideration, the extended periphery of the diaphragm is much longer than the contour of the parallel section of the hull at the point where the diaphragm is sewed.

For attaching the diaphragms to the hull, it is customary, in order to eliminate folds and wrinkling, after determining the minimum dimensions of the diaphragm, to trace a curve of suitable form pass-

ing through the separate points of the contour of the diaphragm itself and of such length that it is shorter by a certain quantity equal to the peripheral extension of the section of the hull where the diaphragm is to be attached. On this curve is sewed a strip of fabric in the shape of the lateral surface of a cylinder. The width of this strip is equal to the distance between the curves mentioned above, and its free edge, which is equal to the section of the hull, must be sewed to the envelope

The diaphragm must be of light fabric, but slightly impervious to the gas, and cut lengthwise of the fabric. The strips forming the diaphragm are sewed uniformly one to the other, all being of the same width. They form an angle of 45° with the longitudinal diametral vertical plane of the hull.

A short time ago new types of diaphragms were proposed, consisting of two panels of fabric set facing each other, impervious to gas and of greater dimensions than strictly necessary (as mentioned above) so that the pockets thus formed between one compartment and the next, would function as ballonets on being inflated with air. This solution, which is not the result of special exigencies, as, for instance, the possibility of dividing the hull into different parts, is not advisable, notwithstanding its ingeniousness, because it contemplates only relatively small ballonets. On increasing the volume of the ballonets, large masses of gas would be displaced in deflating, which is always very dangerous. Furthermore, the surface of the fabric in contact with hydrogen is enormously increased, consequently the

losses of this gas increase proportionately. Lastly, this solution does not result in a lighter arrangement than the ordinary ballonet and airship with diaphragms.

Suspension Catenaries.

The lifting force of hydrogen, distributed throughout the entire hull according to a given law, produces a part of the tension in the hull fabric. Due to the fact that the lifting force can be utilized only in certain concentrated points, various assembly methods have been studied:

1) Designing the hull so that it will automatically concentrate the lifting force of the gas at certain points;

2) Fitting the hull with a rigid element (keel) or a flexible element (rope held by means of a special fabric panel), which is capable of concentrating the lifting force in two or more points according to the type of construction adopted.

If the element is rigid, it is subject to a bending stress acting on the entire structure or only on certain parts of it. If this part is flexible, the flection, entailed by the special type of suspension in question, is supported by the hull. Said bending stress, however, both in the case of the rigid and flexible elements, must not be confused with that obtaining in the entire suspension system in certain types of airships and which we shall discuss at length in another part of this volume. The bending stress is local for the element acted upon by the series of infinitesimal lifting forces

corresponding to the zone of the hull to which the element in question is referred.

The effects of this bending stress often assume big values, while sometimes they are entirely negligible. This depends on the type of suspension adopted.

The first system concentrates, independently of the type of suspension, the lifting force at certain points of the hull and does not produce secondary stresses.

This system is well adapted both for an ordinary bilateral suspension and for a central suspension, as discussed in the preceding chapter. However, for the last mentioned, certain longitudinal stresses in the direction of the hull axis, are not present and consequently the problem is still further simplified.

We shall consider, in the projection of the inflated hull on a vertical plane passing through its longitudinal axis, an element the base of which is dx and on which a lifting force df is acting, which force will be the difference between the true lifting force and the weight of the hull for the corresponding elementary zone, said force being applied at the center of the element dx. The df forces are parallel. We shall combine n such consecutive forces by means of a connecting polygon. On the external sides of this polygon we find two forces, f' and f'', the geometrical sum of which is precisely

$$f = \sum_{1}^{n} df.$$

If the connecting polygon is actually constructed with a rope, the

f' and f'' will be found at the extremities of the rope. Should the ndf correspond to one of the zones (r) into which we preventively divided the hull, f'_r and f''_r are the stresses at the ends of the rope. By repeating the process for all the zones of the hull, the sum $f''_r + f'_{r+1}$ is equal to the available force corresponding to every section of the division, which for the sake of simplicity, we shall call a "node."

We note immediately that the $f''_r + f'_{r+1}$ may have any direction, which affects the suspension, as we shall see later, when dealing with the suspension, by producing stresses in the direction of the axis of the hull. A solution, however, may be found so that the horizontal projection of f''_r = horizontal projection of f'_{r+1} in which case a constant compression from bow to stern is encountered, due to the first f' and the last f''.

In the case of lateral suspension, the rope corresponding to the connecting polygons will be in the air. Hence it is expedient, in order to avoid difficulty with regard to impermeability, for the rope to be entirely located in the air chamber, that is, for the series of the vertices of the connecting polygons to be all located under the projection of the meridian where the ballonet is attached. Furthermore, it is well for said points to lie on the projection of a meridian or on a continuous line.

This condition, which annuls equation (87), fully determines the curve of connection, because the positions of the nodes are also fully established. The anti-projection of this curve is what must actually be used on the hull. Under the line of catenary suspensions

there is still a quantity of gas, air or hydrogen, according to the degree of inflation of the ballonet and, therefore, in order that said gas may be contained in the ballonet, the arcs of the catenary suspensions are lined with light, impervious fabric. This fabric, when the airship is under pressure, stretches and lengthens after the manner of a sail, with a consequent increase in the drag of the airship. In order that the increase be minimum, it is necessary to construct the catenaries very low, but this solution involves strong horizontal compressions. Experience and the type of construction will determine the right height for the catenaries.

On observing the hull constructed in this way, we note that the fabric above the meridian embracing the highest points of the connecting curves, is continuous, while the fabric under it is not. Consequently, the axial tension of the bow is directly balanced by that of the stern in the upper part, while this is not the case for the part under the meridian mentioned above. Consequently, the action of the forces dT acting on the form of the connecting curves, must also be taken into account. The value of these forces dT is perfectly determined, since it depends on the dimension ds of the element of the connecting curve, on its height with respect to the lowest point of the hull and on the internal pressure at that point.

We shall now deal with the method of determining the connecting polygon in question. Let us consider the case in which the dT forces are zero. In this case, we are confronted with only the vertical forces having the same sign. A and B (Fig. 5) are assumed

to be two consecutive nodes of the hull and, therefore, to form part of the same zone or compartment. A" B" D C is the loading diagram due to the force f. The forces 1, 2, 3 ......, corresponding to the length of axis dx, are the df mentioned above. Let us connect said forces by means of a funicular polygon p, starting from node A relative to an arbitrary pole P' with a polar distance H. The connecting polygon will pass through a point B' which does not coincide with B. Through P' let us draw the parallel to B A and where it cuts the line of forces (point E) let us draw the parallel to A B', that is, the line E P. On this line we shall find the pole of the connecting polygon. Let us assume η to be the ordinate of polygon p'. In view of the fact that ηH is constant, we could know the value η_1 and consequently the polar distance $H_1 = \frac{\eta H}{\eta_1}$, which solves the problem, if we knew which is the point M for solving the problem, that is, the point of tangency of the unknown polygon with the meridian line. It is often possible to find immediately the position of the point M from the configuration, by referring to polygon p', but, should this prove difficult, the position of the point P is ascertained after one or two attempts.

After determining the connecting polygon, the forces dT could be applied to it and a second polygon could be determined and the operation repeated. In this way, the solution of the problem could be found through successive approximations. Noting, however, that the dT forces are negligible with respect to the df, it is reasonable to assume that the polygon relative to pole P fuly solves the problem

In case the loading diagram is reduced to a rectangle, the curve p becomes a parabola and the tracing is easily accomplished.

It is better to make use of this case for the central part of the hull.

We here call attention, therefore, to the method of determining the extreme tangents of the parabola. After these tangents are known, the construction of the parabola is easy. By hypothesis, the lifting force P available for unit of length, distributed along the arc, is proportional to the horizontal projection of the arc itself (Fig. 6).

By referring the curve to the horizontal line passing through the highest point and to the vertical (positive downward) passing through the same point, we get:

$$x^2 = 2\,ay \tag{88}$$

The tension in any point

$$R = p\sqrt{a^2 + x^2} \tag{89}$$

The tension at the vertex

$$R_o = p\,a$$

which gives the definition of a. Furthermore, we get

$$\cos e = \frac{dx}{ds} = a\sqrt{\frac{1}{a^2 + x^2}} \qquad \sin e = x\sqrt{\frac{1}{a^2 + x^2}}$$

$$Q = R \cos e = p\,a = \text{const.} = R_o$$

$$P = R \sin e = p\,x$$

$$\left.\begin{aligned} \frac{x_a}{x_b} &= \sqrt{\frac{f_a}{f_b}} \\ x_a + x_b &= L \\ 2a &= \frac{x_a^{\,2}}{f_a} \end{aligned}\right. \tag{90}$$

and the problem is fully solved.

As regards the axial tension T, tending to deform the catenaries, we note that, by calling p the minimum average pressure in flight, the tension, corresponding to the section with a radius of y, is:

$$T_1 = \frac{\pi y^2}{2 \pi y} p = \frac{y}{2} p$$

The increment of tension between the sections y_r and $y_r - d$ is, instead,

$$T_2 = \frac{\pi [y_r^2 - (y_r - d)^2]}{2 \pi y_r} p = \frac{p}{2} (2 y_r d - d^2)$$

The greater part of this tension is absorbed by the hull and $\frac{1}{n}$ part by the two catenaries. Consequently, the part which actually deforms the polygon p is:

$$T = \frac{p}{4n} (2 y_r d - d^2) \tag{91}$$

Before leaving this important problem, we would briefly mention the errors made by substituting an arc of a circle for the arc of a parabola described above. Maintaining the symbols of Fig. 6, the equation of the parabola, referred to the preceding axes, will be

$$x^2 = \frac{x_a^2}{f_a} y \qquad x_a > x_b$$

the equation of the circle passing through A and tangent at point O to the axis xx, will be:

$$x_1^2 = \frac{x_a^2 + f_a^2}{f_a} y_1 - y_1^2$$

By calling u the difference between the ordinates y and y_1 corresponding to a value of $x = x_1$, we get

$$u = \frac{F_a}{x_a^2} \left(f_a \, y_1 - y_1^2 \right)$$

the maximum value of which is given by

$$\frac{du}{dy_1} = \frac{f_a}{x_a^2} \left(f_a - 2y_1 \right) = 0$$

that is,

$$y_1 = \frac{f_a}{2}$$

and consequently, the maximum value is

$$u_{max} = \frac{f_a^3}{4 \, x_a^2} \tag{92}$$

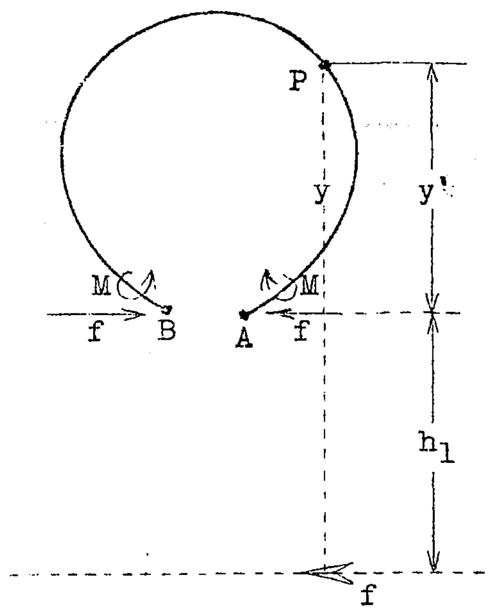

Fig. 1

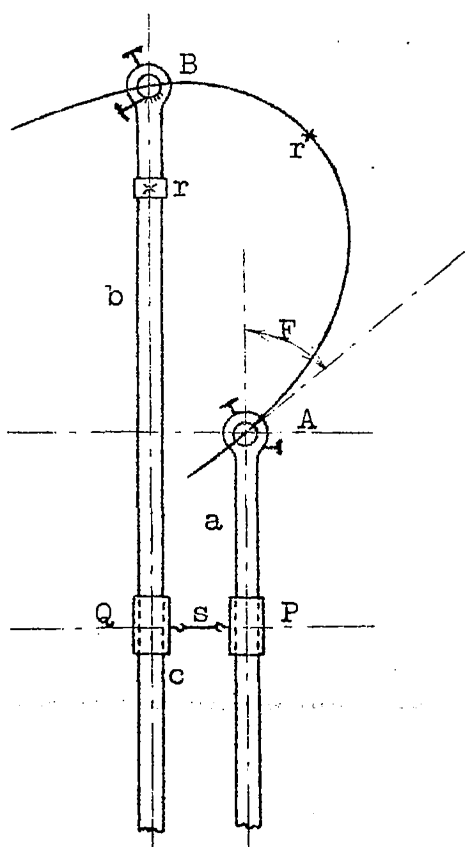

Fig. 2

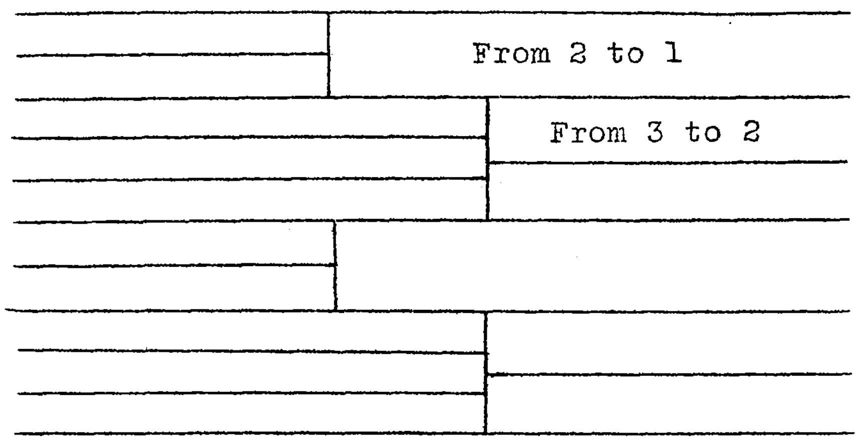
From 2 to 1
From 3 to 2

Fig. 3

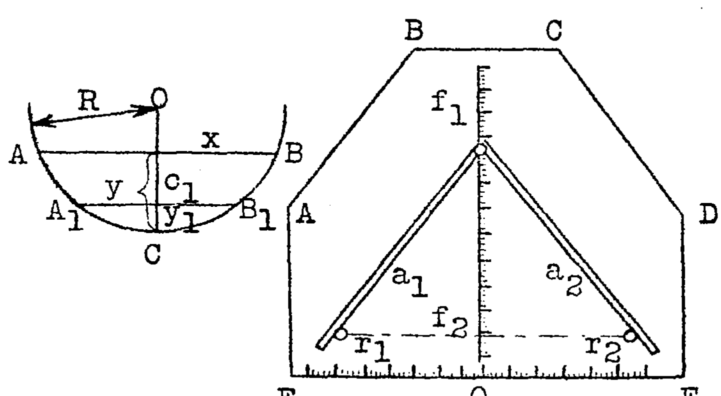
R
O
A
x
B
y
c_1
A_1
y_1
B_1
C
B
C
f_1
A
D
a_1
a_2
f_2
r_1
r_2
F
0
E

Fig. 4

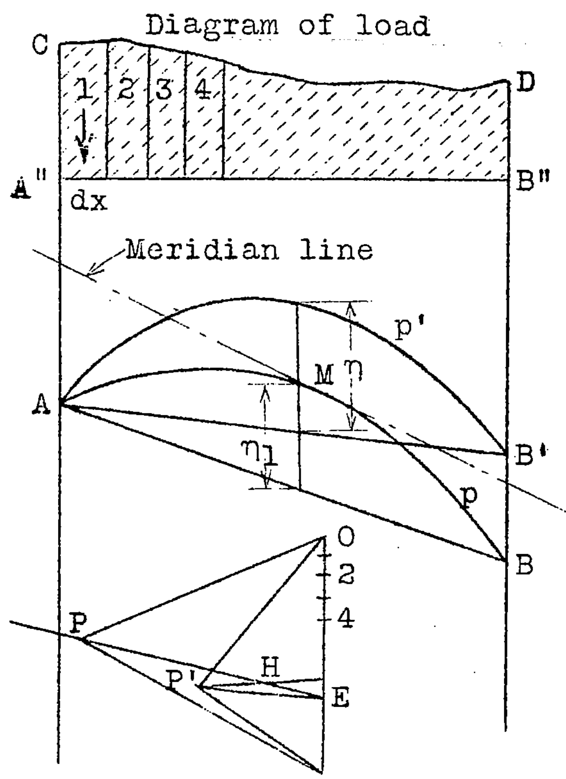
Diagram of load
C
D
1
2
3
4
A"
B"
dx
Meridian line
p'
M
η
A
η1
B'
p
O
2
4
B
P
H
P'
E

Fig. 5

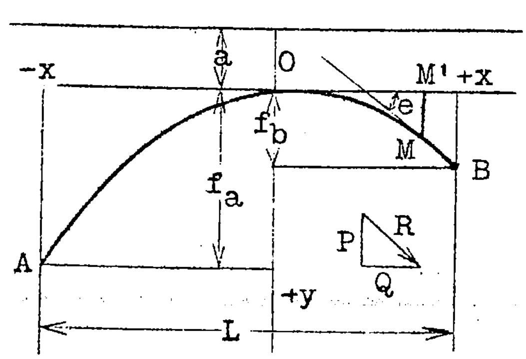
a
O
-x
M' +x
e
fb
M
B
fa
P
R
A
Q
+y
L

Fig. 6